

Catalog Description: A one term stand-alone course on topics in real analysis, covering properties of real numbers, completeness axiom, continuity, convergence of sequences and series of numbers, convergence of sequences and series of functions. Emphasis will be placed on proofs.

Course Objectives: After completing this course, students will be able to

1. State definitions and theorems.
2. Provide examples and counter-examples.
3. Construct mathematically sound proofs.
4. Demonstrate proper use of mathematical language, notation, and symbols.

Learning Outcomes and Performance Criteria

1. Demonstrate a working knowledge of the real number system.

Core Criteria:

- (a) State the field-axioms of the real numbers.
- (b) State the completeness axiom for real numbers.
- (c) Find the least upper bound for a set of real numbers.
- (d) Find the greatest lower bound for a set of real numbers.
- (e) Determine if a point is an accumulation point and/or find an accumulation point.

2. Demonstrate a working knowledge of sequences of real numbers.

Core Criteria:

- (a) Define a sequence of numbers.
- (b) Define the convergence of a sequence of numbers.
- (c) Determine if a sequence is Cauchy.
- (d) Find a convergent subsequence of a non-convergent sequence.

3. Be able to demonstrate a working knowledge of set theory on the real line.

Core Criteria:

- (a) Perform operations on sets (union, intersection, complement, etc).
- (b) Define and give an example of open, closed, finite, infinite, countable, uncountable, and bounded sets.
- (c) Define and give an example of a subset.

Additional Criteria:

- (a) Compute the cartesian product of two or more sets.

4. Demonstrate a working knowledge of functions.

Core Criteria:

- (a) Prove that a function is continuous at a point (epsilon-delta and convergent sequence definitions).
- (b) Identify and classify discontinuities.
- (c) Identify classes of continuous functions (polynomials, sum of continuous functions, etc).
- (d) Determine and verify by proof the limit of function.
- (e) Prove that a function is uniformly continuous at a point.

Additional Criteria:

- (a) Identify the domain and range of a function.
- (b) Show that the inverse image of an open set under a continuous function is open.